Electron distribution functions in laser-embedded plasmas

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A detailed study is reported concerning the evolution of the electron distribution function for a laser-embedded fully ionized plasma, taking into account the inverse bremsstrahlung and the electron-electron (*e-e*) collisions. Whenever possible, an effort is made to provide an analytic treatment of the problem, which is corroborated by numerical analysis as well. An approximate, analytic self-similar distribution function has been found for the case when *e-e* collisions are not negligible, and the heating equation has been solved numerically.

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I. INTRODUCTION

Laser-radiation absorption is an important issue in laser-plasma interactions, in particular as far as plasma heating is concerned. Among the laser-plasma-heating mechanisms, collisional absorption via inverse bremsstrahlung has been extensively investigated for many years [1]. In this subject an important line of investigation has been started by Langdon [2] who showed that for an intense laser field or for a weak field in a plasma with a large ionic charge Z, the electron distribution function differs noticeably from a Maxwellian one and assumes the so-called self-similar form [3] given by $\ln\phi \cong -v^5$. A generalization of these results to the case of higher laser intensities has been given by Jones and Lee [4], who corroborated their findings with Monte Carlo calculations as well. More recently, these results have been supported further by numerical calculations for the case of a uniform plasma [5] and when a steep gradient of temperature is present [6]. Moreover the numerical calculations have shown that the electron distribution function can assume a self-similar form also when the electron-electron collisions are not negligible [5]. These results have a direct consequence on the level of plasma heating which may be achieved with this mechanism and, as a rule, a reduction of the heating is predicted. Very recently Silin and Uryupin [7] have considered the electron scattering on ion-acoustic fluctuations of charge density as another plasma-heating mechanism. The frequency of this collisional mechanism is much higher than that of electron-ion (or electron-electron) collisions and is a highly anisotropic function of the velocity. The investigation predicts an anomalous increase of electron heating in time intervals shorter than the electron-electron (*e-e*) collision time, accompanied by formation of a nonequilibrium electron distribution function. In particular, neglecting electron-electron collisions, a self-similar form of the distribution function is predicted, which is analogous to that first found by Langdon. Other very recent contributions to the subject have been given by Ovchinnikov, Silin, and Uryupin [8] and Chichkov, Shumsky, and Uryupin [9].

Considering the permanent interest for this subject, it is aim of this work to provide an analytical treatment of the electron distribution function for the case when electron-ion and electron-electron collisions are simultaneously accounted for. In Sec. II we give the equation of the evolution of the electron distribution function in the presence of a laser field including the electron collisional terms. In Sec. III we briefly outline the Balescu's theory [3] leading to a self-similar solution when the *e-e* collisions are neglected, and introduce a more appropriate normalization factor. In Sec. IV we extend the treatment outlined in Sec. III to the case when the electron-electron collisions are included. Finally in Sec. V we present an analysis of the evolution of the electron distribution function together with some comments.

II. THE EQUATION FOR THE ELECTRON DISTRIBUTION FUNCTION

Under the condition $\hbar\omega \ll T_e$ (ω being the laser frequency and T_e the electron temperature), there is an excellent agreement between classical and quantum approaches and we will use a well-known classical model [10] to describe a homogeneous plasma with ion charge Z in the presence of a radiation field. The equation for the isotropic part of the electron energy distribution function is written as

$$u^{1/2}\frac{\partial F}{\partial t}=2Y\frac{\partial}{\partial u}\left[\frac{eE^2Z}{3m(\omega^2+\nu_{ei}^2)}\frac{\partial F}{\partial u}+P_1\frac{\partial F}{\partial u}+P_2F\right], \tag{1}$$

where $u=mv^2/2e$, $\nu_{ei}=ZY/u^{3/2}$ is the electron-ion collision frequency with $Y=n_e4\pi(e^2/4\pi\epsilon_0 m)^2(m/2e)^{3/2}\ln\lambda$, where $\ln\lambda$ is the well-known Coulomb logarithm, E stands for the mean-square electric field, and the other symbols have the conventional meaning. The functions P_1 and P_2, which describe the electron-electron collisions are given by

$$P_1(u)=\frac{2}{3}\int_0^u \varepsilon^{3/2}F(\varepsilon)d\varepsilon+\frac{2}{3}u^{3/2}\int_u^\infty F(\varepsilon)d\varepsilon \ , \quad (2)$$

$$P_2(u)=\int_0^u \varepsilon^{1/2}F(\varepsilon)d\varepsilon \ . \quad (3)$$

Since we are not interested to the case of a high anisotropy, we are justified to consider the anisotropic part of the electron distribution function as much smaller than the isotropic one. This assumption restricts the value of the intensity of the applied electric field and forces the drift velocity of the electrons in the external field v_d to be much smaller than the thermal electron velocity v_T. In the case of a high-frequency field the role of the drift velocity is played by the peak velocity of the electron oscillating in the field $v_0=eE/m\omega$, and the use of the two-term expansion of the distribution function is valid when v_0 is considerable smaller than v_T. As far as the inverse bremsstrahlung process is concerned this assumption requires the mean energy gained by an electron in a collision be much less than the thermal energy. Under this condition the motion of the electrons over all the energy spectrum may be described by Eq. (1).

The distribution function satisfies the normalization condition

$$\int_0^\infty \varepsilon^{1/2}F(\varepsilon)d\varepsilon=1 \ . \quad (4)$$

The heating of the plasma is usually described in terms of the electron temperature T_e, which has a precise physical meaning only for a Maxwellian distribution. We will characterize the heating of a non-Maxwellian plasma with the help of an effective electron temperature defined as

$$T_e=\frac{2}{3}\int_0^u \varepsilon^{3/2}F(\varepsilon)d\varepsilon \ . \quad (5)$$

III. INVERSE BREMSSTRAHLUNG HEATING

As it has been shown by many authors [2–6], when the electron-electron collisions may be neglected, due to the inverse bremsstrahlung absorption process the electron distribution function can differ from a Maxwellian. The treatment given by Langdon [2] predicts a self-similar distribution function with the form $\ln\phi\cong -v^5$ [the connection of ϕ to F is displayed below, Eq. (9)]. The *e-e* collisions do not influence the results if

$$\frac{eE^2Z}{3m(\omega^2+\nu_{ei}^2)}\gg P_1(u) \ . \quad (6)$$

This condition may be written in a more convenient way [2], taking into account the fact that $P_1(\infty)\equiv T_e$ as

$$v_0^2/v_T^2\gg 3/Z \ , \quad (7)$$

with $v_T=(eT_e/m)^{1/2}$ the thermal velocity of the electrons. In this section, following Balescu [3], we outline an analytical treatment valid under condition (7) and leading to the self-similar state with a more appropriate choice of the normalization factor.

Under the condition given by Eq. (7), i.e., neglecting the electron-electron terms, Eq. (1) becomes

$$u^{1/2}\frac{\partial F}{\partial t}=2Y\alpha\frac{\partial^2 F}{\partial u^2} \ , \quad (8)$$

where we have introduced the parameter $\alpha=eE^2Z/3m(\omega^2+\nu_{ei}^2)=ZT_e(v_0/v_T)^2/3=\gamma T_e$, with γ a dimensionless parameter. The last equalities of α are specialized to the case when $\omega\gg\nu_{ei}$. This is the case we will consider in our calculations, where the laser frequency will be taken in the optical range. Considering the values of the other parameters of the problem (see, for instance, the caption to Fig. 1), ω will result greater than ν_{ei} by, at least, 4 orders of magnitude.

Following Balescu [3], we introduce a new set of variables

$$t=\tau \ ,$$

$$u=W(\tau)\varepsilon \ ,$$

where $W(\tau)$ is a scaling energy that at the moment is an arbitrary function of time. Owing to the normalization condition (4) we introduce a new distribution function ϕ as

$$F(u,t)=W^{-3/2}(\tau)\phi(\varepsilon,\tau) \ . \quad (9)$$

In the new variables the kinetic equation (8) is then transformed as

$$\frac{\partial\phi}{\partial\tau}-\frac{\dot W}{W}\left[\frac{3}{2}\phi+\varepsilon\frac{\partial\phi}{\partial\varepsilon}\right]=2YW^{-5/2}\varepsilon^{-1/2}\frac{\partial^2\phi}{\partial\varepsilon^2} \ . \quad (10)$$

The above equation admits a self-similar solution if the scaled distribution function ϕ becomes time independent when the relaxation process from the given initial state is over. This means that the distribution function factorizes into a term depending on the time and a term depending only on the energy ε. Dropping the time derivative $\partial\phi/\partial\tau$ as small, Eq. (10) has a time-independent solution when all the coefficients are time independent. Hence, a self-similar solution is obtained if we impose that

$$W^{3/2}\dot W=2Y\alpha B_0 \ , \quad (11)$$

where B_0 is an arbitrary numerical factor. With such an assumption, Eq. (9) becomes

$$\varepsilon^{-1/2}\frac{d}{d\varepsilon}\left[B_0\varepsilon^{3/2}\phi+\frac{d\phi}{d\varepsilon}\right]=0 \ , \quad (12)$$

and its self-similar solution may be written as

$$\phi(\varepsilon)=\phi_0\exp\left[-\tfrac{2}{5}B_0\varepsilon^{5/2}\right] \ . \quad (13)$$

The solution of the self-similar heating equation, Eq. (11), gives the evolution of W as function of the time

$$W_e(\tau)=(W_0^{5/2}+5Y\alpha B_0\tau)^{2/5} \ .$$

Calculating the value of the averaged kinetic energy in terms of the distribution function (9) we get the relation

$$\langle \mathcal{E}_{\rm KE}\rangle=\tfrac{3}{2}T_e=W(\tau)\int_0^\infty\phi(\varepsilon)\epsilon^{3/2}d\varepsilon=W(\tau)\langle\varepsilon\rangle \ ,$$

showing that the function W has the physical meaning of a scaling energy proportional to the electron temperature. From the expression of the self-similar distribution function equation (13) we see that the constant of proportionality depends on the value chosen for the arbitrary parameter B_0. Usually the value of B_0 is taken equal to 5 (see Ref. [3]). We choose instead the value of this parameter so that the function $W(\tau)$ is exactly equal to the electron temperature. This is done putting $\langle \varepsilon \rangle = \frac{3}{2}$ in the above equation yielding

$$B_0 = \tfrac{2}{3}\phi_0 ,$$

and using the normalization condition for the self-similar distribution (13) we get

$$1 = \int_0^{\infty} \varepsilon^{1/2}\phi(\varepsilon)d\varepsilon = \left[\frac{2}{3}B_0\right]^{-3/5} \phi_0 \int_0^{\infty} x^{1/2}\exp(-x^{5/2})dx \approx \phi_0 B_0^{-3/5}\left[\frac{2}{5}\right]^{2/5}\Gamma\left[\frac{3}{5}\right] .$$

From the last two expressions one finds

$$B_0 = 0.3353 .$$

With such a choice, the solution of the self-similar heating equation gives the evolution of the electron temperature as function of the time as

$$T_e(\tau) = (T_0^{5/2} + 5Y\alpha B_0 \tau)^{2/5} . \quad (14)$$

When electron-electron collisions are negligible the solution of Eq. (1), after the relaxation time, becomes close to the self-similar function (13) independently of the form of the initial state. In Fig. 1 we give the effective electron temperature as a function of time for $Z=1$ and 10. Calculations are carried out for a value of the electric field $E=1.5\times10^9$ V/cm, a laser photon energy $\hbar\omega=1$ eV, and an electron density of $n_e=10^{16}$ cm^{-3}. The broken line is referred to an initial δ-function distribution, while the solid one to an initial Maxwellian distribution. The duration of the relaxation time depends on both the shape of the initial distribution function and the values assumed by the quantity $Y\alpha$. In Fig. 1 crosses on the curves mark conventionally the separation of the evolution of the distribution function into two different physical stages. The portion of the curves to the left of the crosses represents the stage of relaxation when the quantity $\frac{2}{3}F_0 T_e^{3/2}$ changes significantly, while the portion on the right side represents the setting of the self-similarity stage, where the same quantity remains constant near the value of B_0 (with a discrepancy smaller than 10%). It is observed than when the initial distribution function is a Maxwellian, the quantity $\frac{2}{3}F_0 T_e^{3/2}$ approaches B_0 from above, while when it is a δ-like function, it approaches B_0 from below.

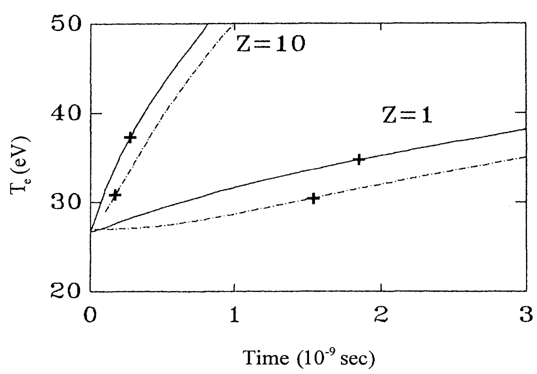

FIG. 1. Electron temperature as function of the time for two different values of the ion charge Z. The solid curves are for the case of an initial Maxwellian distribution, the dashed curves for an initial δ-function-shaped distribution ($E=1.5\times10^9$ V/m, $n_e=10^{16}$ cm^{-3}, $\hbar\omega=1$ eV); crosses on the curves mark conventionally the separation of the evolution of the distribution function into two different physical stages. The portion of the curves on the left of the crosses represents the stage of relaxation when the quantity $\frac{2}{3}F_0 T_e^{3/2}$ changes significantly, while the portion on the right side represents the setting of the self-similarity stage, where the same quantity remains constant near the value of B_0 (with a discrepancy smaller than 10%).

From the reported calculations we see that there is a time delay in the heating of the plasma when the initial electron distribution is assumed to be a δ function. This is due to the existence of two opposite electron flows: (i) the diffusion flow and (ii) the accelerated flow caused by the inverse bremsstrahlung absorption. These two flows compensate each other. For a δ distribution function, F_0 is nearly equal to zero at the beginning and hence the heating is also very small until, during the evolution, the diffusion starts to decrease and does not prevent any more the increasing of the thermal energy of the electron. When the relaxation is complete the heating rate becomes independent from the shape of the initial distribution, a self-similar state is reached and the evolution of the electron temperature as function of the time is given by Eq. (14). Figure 1 shows that the duration of the relaxation time is proportional to Z^{-1}, which is a plausible result considering the process under study and the role of ionic component in it.

IV. INCLUSION OF ELECTRON-ELECTRON COLLISIONS

In this section we consider the case when the electron-electron cannot be neglected. Solving numerically the kinetic equation (1) we have shown that also in this case, after a relaxation time, the product $T_e^{3/2}F_0(t)$ becomes nearly constant indicating the existence of an approximate self-similar solution. Figure 2 shows the value of $T_e^{3/2}F_0(t)$ as a function of the time for two values of the ion charge Z; for both curves the initial distribution is assumed to be a δ function with an energy of 40 eV and the laser parameters give a ratio $v_0^2/v_T^2 \cong 0.085$. With such a choice the electron-electron collisions are not negligible in both cases. We assume now that the evolution of the electron temperature with the time is still described by the heating equation (14) but with a different value of the parameter B_0 which, for the present case, we denote as B. The existence of a self-similar solution implies that the derivative $\partial\phi/\partial\tau$ is much smaller than the other terms in Eq. (1). With such an assumption, applying to the kinetic equation (1) the same transformations given in Sec. III we get

$$\gamma\left[B\varepsilon^{3/2}\phi+\frac{d\phi}{d\varepsilon}\right]+\left[P_2(\varepsilon)\phi+P_1(\varepsilon)\frac{d\phi}{d\varepsilon}\right]=0\ . \quad (15)$$

Imposing a self-similar solution we obtain the heating equation

$$T^{3/2}\dot{T}=2Y\alpha B \quad (16)$$

in the same form as Eq. (11) and the relation

$$B=\tfrac{2}{3}\phi_0\ . \quad (17)$$

The solution of Eq. (15) gives only an approximate self-similar distribution function because its coefficients γ and B depend on time. Therefore the shape of the function ϕ will also depend on time. This approach is valid if the rate of change of the above coefficients with time is smaller than the rate of formation of the approximate self-similar function ϕ. In this case the shape of the self-similar distribution function is given only by the coefficients γ and B depending on the time and the time dependence of the function ϕ will be parametric.

Equation (15) is more general than Eq. (12) in that it gives the self-similar distribution (13) in the limit of small electron temperatures or in other words when condition (7) is fulfilled. On the other hand, for large electron temperatures (small γ) Eq. (15) reduces to

$$\left[P_2(\varepsilon)\phi+P_1(\varepsilon)\frac{d\phi}{d\varepsilon}\right]=0\ ,$$

admitting a Maxwellian distribution as a solution.

Since the parameter $\gamma=\alpha/T_e$ decreases with the time, due to the heating of the electrons caused by the laser field, we conclude saying that the distribution function in its evolution approaches a Maxwellian one and that the characteristic time of this process is the heating time and not the characteristic time of the *e-e* collisions.

We are not able to solve exactly Eq. (15) but we can give a fairly good approximate analytical solution to it. Equation (15) contains the undefined parameter B that must be found self-consistently in order that the integrals of the self-similar functions $\phi(\varepsilon)$ assume the values $P_1(\infty)=1$ and $P_2(\infty)=1$. Numerical calculations of Eqs. (15) and (17) give the relation between B and the γ parameter. The results reported in Table I show that $B\rightarrow B_0$ when $\gamma\rightarrow\infty$. The calculated values may be approximated very well by the equation

$$B=B_0+\left[\frac{0.016}{\gamma+0.06}\right]^{4/5}\ . \quad (18)$$

In Fig. 3 we compare the electron temperature calculated using the exact kinetic equation (1) (solid lines) with the values obtained from Eq. (14) (circles) with B given by Eq. (18). The dashed lines give the heating rate when the *e-e* collisions are neglected. The initial state for all the curves is taken to be Maxwellian. Assuming instead for the initial states a δ distribution function the time τ in Eq. (14) should be replaced by $t-t_R$ where t_R is the relaxation time to the self-similar distribution.

Using the transformation $S=\ln\phi$, Eq. (15) becomes

$$\frac{ds}{d\varepsilon}=-\frac{P_2(\varepsilon)+\gamma B\varepsilon^{3/2}}{P_1(\varepsilon)+\gamma}\ . \quad (19)$$

In the low-energy limit $\varepsilon\rightarrow 0$ the solution of Eq. (19) assumes the asymptotic form

$$S=S_0-\frac{\frac{2}{5}(1+\gamma)B\varepsilon^{5/2}}{\gamma+\frac{2}{5}B\varepsilon^{3/2}}\ . \quad (20)$$

TABLE I. Results of the numerical calculation of Eqs. (15) and (17), giving the relation between the parameters B and γ. ϕ_0 is the maximum value of the self-similar distribution function.

γ	B	ϕ_0
100	0.3357	0.5035
10	0.3396	0.5093
1	0.3706	0.5559
0.5	0.3955	0.5932
0.2	0.4445	0.6667
0.1	0.4923	0.7384
0.08	0.5090	0.7634
0.05	0.5449	0.8174
0.2	0.6120	0.9180
0.01	0.6546	0.9819
0.00		1.0000

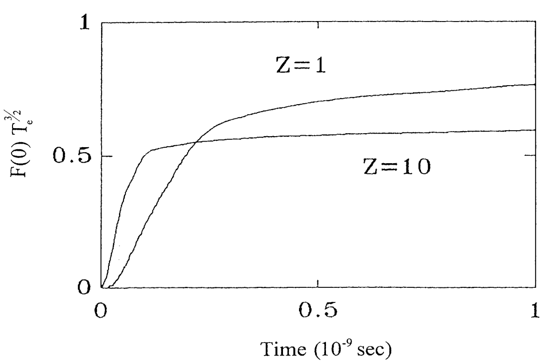

FIG. 2. The parameter $F_0(t)T_e^{3/2}$ as a function of the time for two different values of the ion charge Z. For both curves the initial distribution function is δ-function shaped with the energy of 40 eV and the ratio $(v_0/v_T)^2\cong 0.085$.

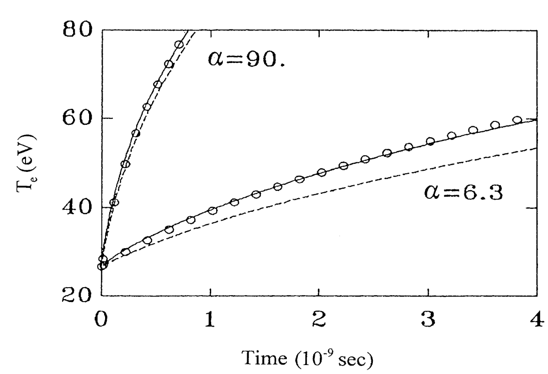

FIG. 3. Electron temperatures as a function of the time for two different values of the parameter $\alpha=eE^2Z/2m(\omega^2+\nu_{ei}^2)=ZT_e(v_0/v_T)^2/3=\gamma T_e$. Solid lines denote the numerical solution of Eq. (1); circles, calculations using Eq. (14), with B given by Eq. (18); dashed lines, calculations neglecting *e-e* collision, Eq. (14).

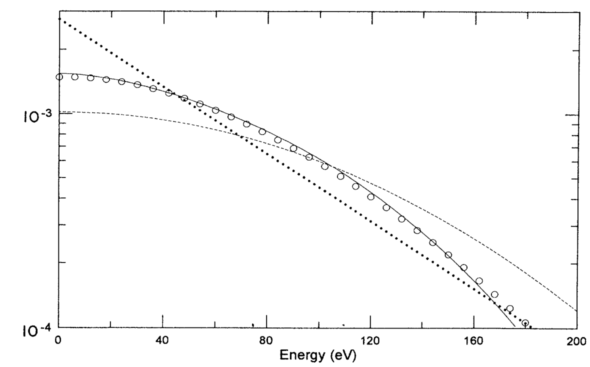

FIG. 4. Comparison of the electron distribution function as given by the analytical formula equation (22) (solid line) and the numerical solution of Eq. (1) (circles); $\gamma=0.4$, $Z=10$, $E=1.5\times10^{9}$ V/cm, $t=10^{-9}$ sec, $T_e=55.2$ eV, $B=0.406$. For completeness, also plotted are the self-similar solution, Eq. (13) (dashed line), and the Maxwellian one (dots).

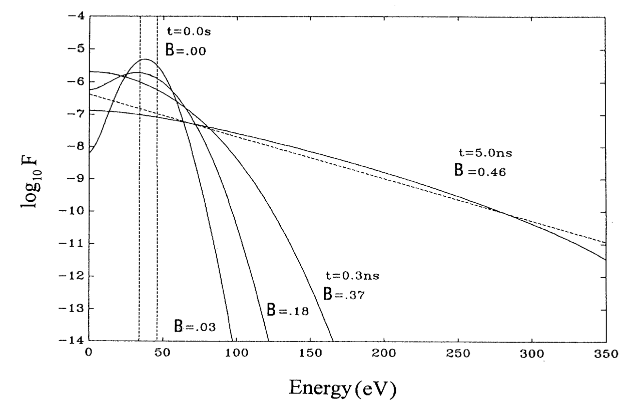

FIG. 5. Electron distribution function vs the energy for different times and values of B [Eq. (17)]. The double vertical dashed lines represent the initial δ-function distribution; the dashed curve is the Maxwellian distribution function. ($E=3.5\times10^{9}$ V/cm, $\hbar\omega=1$ eV, $n_e=10^{16}$ cm^{-3}.)

The basic difference from the distribution function (13) is due to the inclusion of the effect of the e-e collisions. In the other limit $\varepsilon\rightarrow\infty$ the asymptotic solution of Eq. (19) takes the form

$$S=-\frac{\varepsilon+\frac{2}{5}\gamma B\varepsilon^{5/2}}{\gamma+1}\ . \tag{21}$$

Both expressions (20) and (21) reduce to the Maxwellian distribution for $\gamma=0$ and to the Langdon one for $\gamma\gg1$. Combining the two above expressions we obtain

$$\ln(\phi/\phi_0)=-\frac{\varepsilon^{5/2}+\dfrac{2}{5}\gamma B\varepsilon^{4}}{(1+\gamma)\varepsilon^{3/2}+\dfrac{5}{2}\dfrac{\gamma}{B(1+\gamma)}}\ . \tag{22}$$

Equation (22) is one of the main results of this work.

In Fig. 4 we report calculations helping to appreciate the accuracy of Eq. (22). In Fig. 4 we compare the results given by the analytical formula (22) with those given by the numerical solution of Eq. (1), at the value $\gamma=0.4$, which is representative of an intermediate case. For such a value of γ the actual electron distribution function is expected to significantly depart from both a Maxwellian one and the self-similar solution, Eq. (13). In fact, at the chosen value of γ, e-e collisions appreciably modify the self-similar solution, Eq. (13), but are not yet able to make the shape of the distribution function Maxwellian. Other parameters of Fig. 4 are $Z=10$, $E=1.5\times10^{9}$ V/cm, $t=10^{-9}$ sec, $T_e=55.2$ eV, and $B=0.406$. We observe that at the chosen time, the stage of relaxation has been completed, as it is witnessed by the value of B, which is near the value $B(\gamma)$ of Table I.

V. THE EVOLUTION OF THE ELECTRON DISTRIBUTION FUNCTION

We have carried out a detailed analysis of the evolution of the electron energy distribution function. In Fig. 5 we present typical examples of evolution of the electron distribution function during plasma heating by an intense laser. The values of the parameters are taken as before (see Fig. 1); with these values $\gamma\cong0.46$; the initial distribution function is assumed to be a δ function.

Because of the collisions the electron distribution function begins to spread over all the energy spectrum and consequently the product $B=\frac{2}{3}T_e^{3/2}F_0(t)$ changes significantly from zero to the value $B\cong B(\gamma)$, Eq. (18). The evolution process during this time (relaxation time) has an explicit nonstationary character determined by the plasma parameters and by the shape of the initial state. For more intense fields or for larger Z, that is for a larger value of α the relaxation time decreases. When the relaxation is over the distribution function comes close to some self-similar state and the value of the parameter B approaches that given in Table I. In this self-similar regime the change of the distribution function depends entirely on the increment of the electron temperature and the parameter B is determined solely by γ (Table I). The agreement between the actual value of B and that given by Table I may be assumed as a criterion for the setting up of the self-similar state in the course of the evolution of the electron distribution function. With the further increase of the mean electron energy the value of γ decreases (B increases) and the distribution function approaches a Maxwellian shape. This evolution picture is expected to hold true also for more general situations. In conclusion, if at the initial time, for a given value of the laser field and of the ion charge the electron-electron collisions are negligible ($\gamma\gg1$), the formation of a self-similar distribution and its evolution proceed as it has been shown by Langdon [2], but thanks to the heating process the influence of the e-e collisions rises until the self-similar solution transforms from $\ln\phi\cong-\varepsilon^{5/2}$ through the intermediate form Eq. (22) into the Maxwellian distribution.

A final comment concerns a very recent work by Chichkov, Shumsky, and Uryupin [9] who have considered the problem of nonstationary electron distribution functions in a laser field under several conditions, including *e-e* collisions as well. In particular, when $Zv_0^2 \gg v_T^2 \gg v_0^2$ and $Z \gg 1$, Eq. (15) of Ref. [9] gives a distribution function, which amounts to a Langdon-type self-similar function modified by a small correction. Accordingly, it is unable to describe strong departures from Langdon's self-similar function, contrarily to our Eq. (22).

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